Measurement of $L_{eq}$ Using an Integrating SLM

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**Question:** What is an $L_{eq}$ measurement when using an Integrating Sound Level Meter to measure sound level?

**Answer:** The $L_{eq}$ is the level of a constant sound over a specific time interval that has the same sound energy as the actual varying sound over the same interval. To better understand the meaning of $L_{eq}$, let’s start from the beginning and discuss how sound and sound levels are measured using an Integrating Sound Level Meter.

Sound is made up of rapid oscillatory compressional changes in a medium, such as air, that propagate over a distance. These changes can be characterized by changes in the density, pressure, and motion of air molecules. The pressure fluctuations are detected by the human ear and transformed to signals that the brain interprets. Our perception of sound depends largely on a sound's level, frequency content and time varying characteristics.

The transducer used in a Sound Level Meter (SLM) is a microphone. This is a sensor that converts dynamic pressure variations into electrical voltages. The Sound Level Meter then processes these analog signals and computes and displays a variety of descriptive parameters about the measured sound. There are literally dozens of descriptive parameters that can be used to quantitatively characterize the measured sound. Which one you use will depend on the purpose of the measurement. For instance, you may measure sound levels for a community noise standard, occupational noise exposure evaluation, product noise exposure, etc. This article will only focus on a few of the many measurements that Sound Level Meters can provide.

An SLM is designed to give objective, reproducible measurements of sound pressure level or sound level. Figure 1 shows a 4-second time history of sound pressure fluctuations measured with a SLM positioned 1 meter in front of a loudspeaker. The sound source originates from one of my favorite music tracks, The Right Thing (at the Wrong Time), from the Rong CD by Quiltedfish Records. The recorded measurement is calibrated in pressure units of Pascals. One Pascal [Pa] is equal to one Newton per meter squared or $1.45 \times 10^{-2}$ pounds per square inch [psi]. From Figure 1 we see that the pressure fluctuations vary around zero. Actually, however, the pressure fluctuations vary about the static atmospheric pressure (~101,320 Pa). Since we generally use microphones that are vented (resulting in a low frequency cutoff of 5-20 Hz) we only measure the dynamic component of the pressure fluctuations. From Figure 1 we see that it is difficult to describe the magnitude of the sound because the instantaneous pressure level is constantly changing over the 4-second time frame that was recorded.

SLMs can also weight the measured signal as a function of frequency before computing any descriptive parameters for the measured sound. The purpose of this weighting network is to simulate how the sensitivity of the human ear varies with the frequency of sound. The standard types of networks available for most SLMs are 'A' and 'C' weighting. The 'A' network weights a signal in a manner that approximates an inverted equal loudness contour of human sensitivity to pure tones at low sound pressure levels. The ‘C’ network weights a signal in a manner that approximates an inverted equal loudness contour for human sensitivity to pure tones at high sound pressure levels. In addition to these frequency weightings, most SLMs have an 'L' (linear or overall) setting that does not weight the measured signal. All the measurements in this article have been made without any weighting thus they are all linear or unweighted.

Once the signal has been weighted, the resultant signal is passed through a circuit or calculation to determine its Root Mean Square (RMS) value. The RMS value is very important in sound measurement because it is directly related to the energy of the sound being measured. In the past, SLMs were analog instruments and used RMS detection circuits to measure the RMS level. Measured RMS values were then displayed using an analog meter. Since analog meters were typically electromechanical devices, they were limited to how fast they could keep up with rapidly fluctuating sounds making it difficult to accurately measure the level. Because of this problem, meters were standardized on two detector response characteristics: 'F' for Fast and 'S' for Slow. The Fast time constant is 125 ms whereas the Slow time constant is 1 second. Thus, the Fast detector would provide for a fast reacting display allowing the meter to display a sound level that was not changing too rapidly. The Slow detector would give an even slower response thereby letting the analog meter display levels that otherwise would be difficult to read using the Fast time constant.

Today’s SLMs use digital displays which no longer have the response limitations of the analog meter. However, they would still be difficult to read if the RMS values being displayed were updated too rapidly. Because of this, most SLMs display the maximum RMS value within the preceding second. The detector response setting of Fast or Slow now really just depends upon the standard to be followed.

The RMS detectors in most SLMs use an exponential, moving average technique that is continually accepting new data. The equation below illustrates the method:

$$P_{RMS} = \frac{1}{\tau} \int_{t}^{t+\tau} p(t)^{2}dt$$

In the above equation the sound pressure $p(t)$ (see Figure 1) is squared and multiplied by an exponential decay factor. When the time of integration is lesser than the current time $t$ the value of the pressure squared is essentially undiminished. For times less than the current time, the value is diminished exponentially so it becomes less important. The constant $T$ expresses the rate at which the older data become less influential. The larger the value of $T$ the slower the decay factor reduces and the slower the response of the system to rapid changes. The values of $T$ are the standardized values of Fast (125 ms) and Slow (1000 ms) as discussed above. Figure 2 shows the computed RMS sound pressure overlaid on the measured sound pressure. Notice that the RMS sound pressure is always a positive value and lower in amplitude than the peaks of the instantaneous measured sound pressure.

When dealing with sound pressure we rarely use units of Pascals because of the extremely large dynamic range of pressures that the human ear is capable of detecting. The minimum sound pressure that the ear can detect is 20 μPa. The threshold of pain is 200 Pa, which is 10,000,000 times larger than the threshold of hearing. So, when dealing with sound pressures, we generally express them as the logarithmic ratio of a measured pressure to a reference pressure using decibel (dB) notation. The decibel scale allows a range of sound pressures from say 0.000002 to 200 Pa to be expressed on a linearly graduated scale of –20 to 140 dB relative to a reference pressure:

$$L_{p} = 20 \log_{10} \left( \frac{P_{RMS}}{P_{0}} \right)$$

From the above equation we see that the decibel scale for sound pressures is based on the ratio of a measured value above, equal to or below a reference value $P_{0}$. In airborne acoustic measurements, we use the threshold of hearing as the reference value; that is $P_{0} = 20 \mu Pa$. The equation for Sound Pressure Level (SPL) in dB re 20 μPa becomes:

$$L_{p} = 20 \log_{10} \left( \frac{P_{RMS}}{20 \times 10^{5} Pa} \right) = SPL, dB$$
The threshold of hearing is 0 dB SPL and the threshold of pain is 140 dB SPL, a much more manageable range of values. If the sound pressure levels had been ‘A’ weighted, the measured values would be specified in terms of Sound Levels in dBA. The A-weighted sound level scale provides a much better estimate of hearing damage risk in industry and annoyance to noise compared to an overall or linear measurement of sound pressure level. Figure 3 shows the RMS pressure values from Figure 2 in terms of SPL in dB.

If I were to ask you to quantify the SPL over the 4-second period of the measurement shown in Figure 3, you would have difficulty because the SPL is constantly changing. And indeed most of the SPL measurements you make would show this type of variability and perhaps more.

Because of the variability in sound levels for one or more sources, a descriptive measurement parameter was developed to reduce the sound level over an interval in time to a single equivalent value. This value is the $L_{eq}$ or equivalent continuous sound level. This can be related to a constant sound over a specific time interval that has the same acoustic energy as the actual varying sound over the same interval. If the sound source being measured emitted a constant fixed tone, the $L_{eq}$ value would simply be the same as the sound level. But what happens when the sound level varies like that in Figure 3? The $L_{eq}$ is calculated using the following equation:

$$L_{eq} = 10 \log_{10} \left[ \frac{1}{T} \int_0^T \left( \frac{p(t)}{p_0} \right)^2 dt \right]$$

where:
- $p(t)$ = measured sound pressure, Pa
- $p_0$ = reference pressure, 20 $\mu$Pa
- $T$ = specified measurement duration time, sec

The $L_{eq}$ is a logarithmic average of the SL over a specified time interval T. Figure 4 shows the original SL measurements with the equivalent continuous sound level $L_{eq}$ (83.5 dB) superimposed.

Two other descriptive parameters are also shown in Figure 4, the $L_{max}$ and $L_{min}$. The $L_{max}$ parameter is the maximum SL that occurred in the measurement interval T. The $L_{min}$ is the minimum SL that occurred in the measurement interval T. The three parameters $L_{eq}$, $L_{max}$, and $L_{min}$ together effectively describe the sound level characteristics over the measurement interval T. An Integrating Sound Level Meter is required to calculate an $L_{eq}$ value. Some SLMs give the user complete flexibility in setting up the measurement interval T while others only have a pre-defined set of choices for the measurement interval. Not all Integrating Sound Level Meters are the same as for the descriptive parameters they calculate. For example one type of meter may only display the $L_{eq}$ value where another one may display the $L_{eq}$, $L_{max}$, and $L_{min}$ along with a host of others. Many of the standards and ordinances used for sound level measurements are based upon the $L_{eq}$ value measured over some specified time interval.

Another descriptive parameter that is sometimes used as an alternative to the $L_{eq}$ value for short duration, high level sound sources is the Sound Exposure Level (SEL). This parameter is defined as a constant sound level lasting for one second that has the same amount of acoustic energy as the original sound. This parameter is useful for comparing different types of impulsive or short duration noise events. For the 4-second measurements shown in Figure 3 the $L_{SEL}$ = 89.5 dB. The equation that defines the $L_{SEL}$ is shown below and is similar to the equation for $L_{eq}$ except that it is normalized to 1 second instead of the measurement interval T.

$$L_{SEL} = 10 \log_{10} \left[ \frac{1}{T} \int_0^T \left( \frac{p(t)}{p_0} \right)^2 dt \right]$$

When your measurements require more detailed information regarding the sound level over a specified time interval than what the $L_{eq}$, $L_{max}$, and $L_{min}$ parameters provide, there is another set of descriptive statistical parameters available with some SLMs. These parameters are called $L_{10}$ or Percentile Levels and are based upon the cumulative distribution of sound levels over a specified time interval. For example, the $L_{10}$ value is the SL that is exceeded only 10% of the time. Figure 5 illustrates the cumulative distribution of the SL of the 4-second measurement shown in Figure 3 and highlights the $L_{10}$ and $L_{90}$ values.

In this Q&A column we have described some of the fundamental sound pressure level and sound level parameters that are measured and calculated using Integrating Sound Level Meters. In some sense we have only scratched the surface since there are many other descriptive parameters that can be used to help quantify a sound’s level, frequency content and time varying nature. Determine which parameters are required for your specific measurement circumstances and what measurements are available in a specific SLM before selecting an Integrating Sound Level Meter.

Next Months Question: Which Signal Processing Window should I be using to analyze my data?